The Singularity in General Relativity

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Abstract

General relativity (GR) is characterized by natural singularity. In this approach, it is demonstrated that there are two different kinds of Einsteinian singularities. If the divergent term in the Kreshmann scalar has a cutoff, the singularity will be finite and discrete will occur automatically near the singularity based on the general shape of the singularity.

1 Introduction

General relativity is characterized by natural singularity. There are some "rough edges" in some solutions to the Einstein equation, known as space-time singularities, and time-space can be investigated along the path of light. However, the geodesic is abruptly broken, therefore the singularity cannot be geometrically characterized. The fact that there may be more than one point at the singularity is important. Furthermore, these "rough edges" of field equation solutions can be either a ring or a point. The Kerr black hole, for instance, has an odd number of loop singularities. In the singularity of space-time's curvature, it is limitless [1]. Nature does not possess singularity. The motionless Schwarzschild black hole is a well-known illustration of the singularity's (the end of time's) future[2]. The universe had a past singularity (the beginning of time), sometimes known as the big bang, according to the Friedmann-Lemaitre-Robertson-Walker (FLRW) gauge [3].

It is simple to speculate whether this is an unnatural byproduct of the formation of singularities because these solutions are maximally symmetric. Singularities, according to the well-known sin-

gularity theorem, are universal characteristics of general relativity. The mass will collapse and reach the special point state [4], which is a singularity [5], as long as the mass is large enough. The nature of the singularity, however, remains a mystery about singularities and is the focus of the most recent study. The singularity theorem does not address this issue. On the other hand, it is thought to be superior to the naked singularity for the singularity enclosed by the horizon. According to the cosmic oversight hypothesis, the event horizon will cover all genuine future singularities, preventing the outside world from seeing any evidence of the singularity state. Although there is no concrete evidence in favor of this, simulation findings show it to be true.

In this study, we demonstrate that, depending on the type of singularity, there are two types of singularities in all general relativity solutions. If the divergent term has a cutoff, the singularity will be finite and automatically quantized based on its general form.

2 The Singularity in General Relativity

2.1 The Schwarzschild Black Hole

$$ds^2 = -(1 - \frac{2M}{r})dt^2 + (1 - \frac{2M}{r})^{-1}dr^2 + r^2d\theta^2 + r^2sin^2\theta d\varphi^2$$

The Kreshmann scalar as follows

$$\mathcal{H} = \frac{48M^2}{r^6}$$

2.2 The Kerr-Newman Black Hole

$$ds^2 = -(1 - \frac{2Mr - Q^2}{\rho^2})dt^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2d\theta^2$$
$$+ [(r^2 + a^2)sin^2\theta + \frac{(2Mr - Q^2)a^2sin^4\theta}{\rho^2}]d\varphi^2 - \frac{2(2Mr - Q^2)a \cdot sin^2\theta}{\rho^2}dtd\varphi$$

where $\rho^2 = r^2 + a^2cos^2\theta$, and $\Delta = r^2 - 2Mr + a^2 + Q^2$. The Kreshmann scalar as follow

$$\mathcal{H} = \frac{48M^2(r^6 - 15a^2r^4cos^2\theta + 15a^4r^2cos^4\theta - a^6cos^6\theta)}{(r^2 + a^2cos^2\theta)^6}$$

2

And when radius is sat in metaphase plane $\theta = \pi/2$, the Kreshmann scalar as follow

$$\mathcal{H}|_{\theta=\pi/2} = \frac{48M^2}{r^6}$$

As a conclusion, the kerr-Newman black hole is a general form of other special black hole, such as Kerr black hole so on

2.3 The Friedmann-Lemaitre-Robertson-Walker(FLRW) Gauge

$$ds^2 = -\frac{1}{a^2}dt^2 + [\frac{1}{1-kr^2}dr^2 + r^2d\theta^2 + r^2sin^2\theta d\varphi^2]$$

The Kreshmann scalar in FLRW gauge

$$\mathcal{H}' = \frac{12[a^2\ddot{a}^2 + (\dot{a}+k)^2]}{a^4}$$

When $a \to 0$, the Kreshmann scalar as follows

$$\mathcal{H}'|_{a\to0} = \frac{12k^2}{a^4}$$

3 Analysis about classification of singularity

According to Kreshmann scalar of KN black hole

$$\mathcal{H} = \frac{48M^2}{r^6} = \frac{1}{R^6} = \lim_{N\to\infty}\sum_{n=0}^{N}\sum_{m=0}^{n}\frac{(n+5)!}{5!\cdot m!\cdot(n-m)!}(-R)^m$$

According to Kreshmann scalar of FLRW model

$$\mathcal{H}' = \frac{12k^2}{a^4} = \frac{1}{A^4} = \lim_{N\to\infty}\sum_{n=0}^{N}\sum_{m=0}^{n}\frac{(n+3)!}{3!\cdot m!\cdot(n-m)!}(-A)^m$$

where $R = (48M^2)^{-\frac{1}{6}} \cdot r$. When $r \to 0$, there $R \to 0$. And where the condition of series expansion is as follows

$$0 < r < (48M^2)^{\frac{1}{6}}$$

where $A = (12k^2)^{-\frac{1}{4}} \cdot a$. When $a \to 0$, there $A \to 0$. And where the condition of series expansion is as follows

$$0 < a < (12k^2)^{\frac{1}{4}}$$

4 Discussion in General

$$\mathscr{H}|_{r \to 0} = \frac{1}{\mathcal{R}^\upsilon} = \lim_{N \to \infty} \sum_{n=0}^{N} \sum_{m=0}^{n} \frac{(n + (\upsilon - 1))!}{(\upsilon - 1)! \cdot m! \cdot (n - m)!} (-\mathcal{R})^m$$

$$\approx \lim_{N \to \infty} \sum_{n=0}^{N} \frac{(n + (\upsilon - 1))!}{(\upsilon - 1)! \cdot n!} = \lim_{N \to \infty} \frac{(N + \upsilon)!}{\upsilon! \cdot N!}$$

Where $\mathscr{H} = \mathcal{H}$ or $\mathcal{H}'$ and $\mathcal{R} = R$ or A

In the singularity of GR, $N \to \infty$ is a hypothesis. If the value of kreshmann scalar is limited in all space-time and we will find out that kreshmann scalar is discrete automaticly. If there is a cutoff in value of M, the divergent term will be limited as follows

$$\mathscr{H}^M|_{r=0} = \frac{(N + \upsilon)!}{\upsilon! \cdot N!} = \frac{M!}{\upsilon! \cdot (M - \upsilon)!}$$

Where $M = N + \upsilon$. And there M stand for the state number in singularity. The gravitational quantum has mode number with υ. In every bending state (energy level) $\mathscr{H}^M$, there are the state number M in singularity, and those states is occpied by identical gravitational quantum with mode number υ. Every configuration has a uniform contribution for bending state (energy level).

5 Conclusion

The singularity, according to the singularity theorem, is fundamentally connected to the nature of gravity and the structure of GR. The kreshmann scalar divergent in singularity indicates that GR has a limited range of application. This paper analyzes GR's behaviors in singularity and attempts to eliminate the divergent term.

In a conclusion, it is demonstrated that there are two sorts of singularities in all solutions to general relativity. If the singularity's divergence term has a cutoff, the singularity will be finite and automatically quantized based on its general form. This suggests that it works in the general relativity singularity area. And there,

in singularity, is the state number. The mode number of the gravitational quantum. There exist singularities in every bending state, and those singularities are occupied by identical gravitational quantum systems with mode numbers. Each configuration contributes uniformly to the bending state. Acoording to the field equation of GR, bending state is linked with the local energy level of spacetime. So, when $r \to 0$, the energy level become discrete, showing the characteristic of quantum theory.

Singularity in general of GR

$$\mathscr{H} = \frac{1}{\mathcal{R}^v} = \sum_{n=0}^{\infty} \frac{(n+(v-1))!}{(v-1)! \cdot n!}(1-\mathcal{R})^n = \lim_{N\to\infty} \sum_{n=0}^{N} \frac{(n+(v-1))!}{(v-1)! \cdot n!}(1-\mathcal{R})^n$$

$$= \lim_{N\to\infty} \sum_{n=0}^{N} \frac{(n+(v-1))!}{(v-1)! \cdot n!} \sum_{m=0}^{n} \frac{n!}{m! \cdot (n-m)!}(-\mathcal{R})^m$$

$$= \lim_{N\to\infty} \sum_{n=0}^{N} \sum_{m=0}^{n} \frac{(n+(v-1))!}{(v-1)! \cdot m! \cdot (n-m)!}(-\mathcal{R})^m$$

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